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# A METHOD FOR PROBLEM DIAGNOSIS IN MULTIVARIATE QUALITY CONTROL: CONSTRAINED SOLUTION SPACES FOR PROCESS ORIENTED BASIS REPRESENTATIONS

**Burak BİRGÖREN**
Kırıkkale University, Engineering Faculty, Department of Industrial Engineering, Kırıkkale, Turkey

## ABSTRACT

Diagnosis of problem causes has been an important concern in multivariate quality control. Some of the recent research on this subject has taken their motivation from the fact that there are some patterns in multivariate data that can be directly linked with problem causes. Process oriented basis representations is a methodology developed for identifying such patterns; it is based on multiple linear regression. This study proposes some improvements on this method for better identification of patterns. The basic idea is to constrain the solution space using practical engineering bounds.

**Key Words:** Problem diagnosis, Multivariate quality control, Multiple linear regression

## ÇOK DEĞİŞKENLİ KALİTE KONTROLDE PROBLEM TEŞHİSİ İÇİN BİR YÖNTEM: SÜREÇ TABANLI TEMEL GÖSTERİMLERİ İÇİN ÇÖZÜM UZAYININ SINIRLANDIRILMASI

## ÖZET

Çok değişkenli kalite kontrolde problem kaynaklarının teşhisi önemli bir sorundur. Bu konu üzerine yakın zamanda yapılan araştırmaların bir kısmı, çok değişkenli verilerde oluşan bazı desenlerle problem kaynakları arasında doğrudan bağlantılar olmasından hareket etmişlerdir. Süreç tabanlı temel gösterimleri, bu desenleri saptamak amacıyla geliştirilmiş bir yöntemdir; çoklu doğrusal regresyona dayanmaktadır. Temel fikri çözüm uzayını pratik mühendislik sınırları kullanarak kısıtlamaktır.

**Anahtar Kelimeler :** Problem teşhisi, Çok değişkenli kalite kontrol, Çoklu regresyon analizi

## 1. INTRODUCTION

Advances in the sophistication of sensor devices and decreasing costs for computer networks have led to production systems that monitor many characteristics of each product simultaneously, and capture these characteristics in computer databases [1-3]. Multivariate quality databases provide greatly increased information, but they present a challenge to a quality practitioner who wants to use the data effectively. Past research on techniques for multivariate SPC has provided tools to identify when irregularities in production occur, and to characterize, in a multivariate sense, the major components of this variation. However, it is up to a quality practitioner to identify the cause or causes of these irregularities and to determine the appropriate action.

The most popular multivariate SPC technique is based on the Hotelling's $T^2$ statistic [4]. This technique reduces a multivariate observation to a univariate statistic and plots it on a so-called multivariate control chart; the control limits are easily obtained under the assumption of multivariate normality. Other multivariate charts also have been proposed based on derivatives of the Hotelling's $T^2$ statistics [5].

The difficulty of interpreting an out-of-control signal on a multivariate control chart has been discussed extensively [5-9]. A widely accepted view for interpretation is to determine which process variable or variables are responsible for an out-of-control situation. When the Hotelling's $T^2$ statistic is plotted on a multivariate control chart, however, interpretation is rather difficult. An alternative approach is to plot separate univariate charts on each variable. This approach directly indicates the variables that are out of control, but ignores the correlation. It is commented that the combination of using a multivariate chart for signaling purposes, and then using separate univariate charts for diagnostic purposes is often effective [5].

The use of principal components is also recommended to aid the interpretation [6, 9, 10]. It is pointed out that by using both the individual variables and the principal components with the univariate charts, the information about the correlation effect is not lost. In some situations, principle components have physical interpretation; hence, they can be individually used for diagnostic purposes. Recently, a method related to principle component analysis and factor analysis is proposed to describe the relationship between process faults and process variability [11].

Regression models have also been used for diagnostic purposes in multivariate SPC. Hawkins [12, 13] has suggested that regression adjustments of variables may be an effective alternative to classical multivariate control charts when the likely departures from control has a known structure. He proposed handling correlated multivariate normal variables by regressing each variable against all others, and charting the regression residuals. Another application of regression in multivariate SPC is cause-selecting control charts which distinguish controllable assignable causes, uncontrollable assignable causes, and common causes by constructing regression adjusted process variables [14].

It is noted by many researchers that likely departures from an in-control process may have known patterns, and different methods are proposed for identifying these patterns [1, 2, 12, 16-19]. One of these is the Process-Oriented Basis Representations (POBREP) methodology developed by Barton et al. [1]. It is further studied and improved by Gonzales-Barreto [17] and Birgoren [18]. POBREP is based on multiple linear regression; it provides a representation of multivariate data in terms of known patterns. This paper first gives an overview of the methodology, discusses its weaknesses, then proposes a technique for better pattern recognition. The technique achieves an improvement by constraining the solution space in multiple regression, hence called the Constrained Solution Space (CSS) technique.

## 2. POBREP METHODOLOGY

POBREP is a process diagnostic methodology which identifies the most likely causes of bias or variation in product performance by linking patterns in multivariate performance data with patterns associated with certain kinds of production problems. Many manufacturing systems with vision systems or sensor-based inspection capabilities provide repeated measurements of the same quality characteristic over multiple two or three-dimensional locations on a manufactured part. For example in a stencil printing operation in electronics manufacturing, a vision system takes measurements of solder paste volume at several locations along a rectangular region on which an integrated circuit will be mounted later. For a high-pitch component, the number of measurements can exceed 200. For a single part, a multivariate quality vector x is defined as the set of m measured deviations from nominal.

Linking process errors with the resulting pattern of errors over the surface of a manufactured part provides a way to diagnose observed error patterns in such parts. Suppose that it is possible to identify a pattern of errors for each potential cause of process bias or variability. Suppose that k different patterns of interest can be identified for k different process causes, say $a_1, a_2, \ldots, a_k$, where $a_i$'s are m dimensional vectors. If the vectors $a_1, a_2, \ldots, a_k$ are independent and m = k, then the cause related patterns provide an alternative basis for representing the same quality vector, and the representation of x in this basis is $x = z_1a_1 + z_2a_2 + \ldots + z_ka_k$. That is, x can be thought of as a weighted sum of characteristic patterns, where the amount of pattern $a_i$ in x is indicated by the coefficient $z_i$. The vector $z = (z_1, z_2, \ldots, z_k)'$ can be found by solving the system of linear equations

$$x = A\,z, \tag{1}$$

where A is the matrix composed of column vectors $a_1, a_2, \ldots, a_k$:

$$A = [a_1 \mid a_2 \mid \ldots \mid a_k]. \tag{2}$$

This basis, { $a_1$, $a_2$,…, $a_k$ }, is called a process-oriented basis, and each $a_i$ is called a basis element. By decomposing the observed quality vector, x, into patterns corresponding to known causes, i.e. by solving the system of linear equations to find the z vector, a process-oriented basis representation is formed. The components of the z vector, $z_i$, i = 1,…, k, are called POBREP coefficients. Using the process-oriented basis representation z, diagnosis is possible: potential causes are associated with patterns ($a_i$) having large positive or negative POBREP coefficients ($z_i$).

For instance, Gonzalez-Barreto [17] modeled the following four problems in stencil printing operation by four basis elements: poor board alignment on the horizontal axis, poor board alignment on the vertical axis, insufficient paste and squeegee pivot problem. Lack or excess of solder paste volume at each printing location around the rectangular printing area causes serious quality problems, and each of the four problems gives rise to a distinct pattern of deviations in the amount of solder paste around the rectangular area. Gonzalez-Barreto [17] showed that decomposing the multivariate quality vector using these patterns as basis elements aids considerably in the detection of the mentioned problems. Similarly, Birgoren [18] modeled problems in a drilling operation.

Note that in many cases it will not be necessary to construct a complete basis. This corresponds to a situation where k < m. Then, the process-oriented basis may not span the subspace that x lies in, hence there may be no solution to the system of linear equations in Equation 1. In this situation, x can be represented as a linear combination of k basis elements and a residual vector in the following regression equation form:

$$x = A\,z + e \qquad (3)$$

which can be solved by the least squares method. However, the POBREP methodology differs from the traditional regression context in that Equation 3 is solved for many consecutive quality vectors, and this allows analyzing the behavior of z and e over time. Also, statistical properties of POBREP coefficients are different. The probability model in multiple linear regression is

$$X = A\,z_0 + \varepsilon, \qquad (4)$$

where $\varepsilon \sim \mathcal{N}(0, \sigma^2 I)$, I is the identity matrix, 0 is the zero vector, and $z_0$ is a fixed but unknown parameter vector; z coming from the least squares solution of Equation 3 is an estimate of $z_0$. In POBREP context, however, it is more realistic to assume that the components of $z_0$, $z_{0i}$, are themselves random variables, since, in general, problems associated with basis elements will exist in each product with varying degrees. In this case, $z_0$ represents the true levels of problems associated with the basis elements, and z from the solution of Equation 3 will give an estimate for the realizations of $z_0$ in a least squares sense. The POBREP methodology monitors the estimated values $z_i$ in place of $z_{0i}$.

It is reasonable to assume that the randomness associated with each $z_{0i}$ is a result of the common cause randomness in the process problem or problems associated with $z_{0i}$. Also the vector ε, models the common cause variations that do not act on basis elements for the specified process problems. By solving Equation 3, x is represented in terms of the basis elements in a least squares sense where e represents the part of x that cannot be explained by the basis elements. Therefore, e should be considered as an estimate of ε,

The in-control state of a process in POBREP setting can be defined as follows according to the above assumptions: The multivariate quality vector is produced by the probability model in Equation 4 such that each $z_{0i}$ and ε, has a stable distribution. Thus, an in-control process might have POBREP coefficients with stable distributions but with large means and/or large variances. This situation concerns the capability of the process rather than the in-control state of the process.

In addition to monitoring POBREP coefficients using multivariate SPC techniques, Gonzalez-Barreto [1] recommended monitoring the squared norm of the residual vector, $\|e\|^2$, over time for identifying missing basis elements. If the basis elements that form A do not capture the complete structure in the multivariate quality vector x, that is, if there are patterns of process errors that are not represented in A, then these patterns may cause the $\|e\|^2$, values to be dependent; further, these patterns will appear in e when Equation 3 is solved.

## 3. DESIRED PROPERTIES OF PROCESS-ORIENTED BASIS ELEMENTS AND SCALING

There are certain properties that process-oriented basis elements should satisfy in order to maximize the performance of the POBREP methodology. The primary one is that basis elements should be easy to interpret from an engineer's point of view. The engineer who applies the POBREP methodology should be able to understand what kind of error patterns basis elements represent, and also should be able to construct the basis easily. Orthogonality is another desired property, because severe non-orthogonality (or linear near-dependency) may give rise to unreliable POBREP coefficients. This problem will be addressed in the following section. The POBREP methodology uses the least squares method to calculate POBREP coefficients which involves matrix computations. Therefore, the process-oriented basis matrix, $A = [a_1 \mid a_2 \mid \ldots \mid a_k]$, should also be structured such that the numerical errors are minimized. Scaling of process-oriented basis elements solves the problems associated with ease of interpretation and numerical errors to a satisfactory degree.

Before discussing scaling of process-oriented basis elements, we provide a closer look at how patterns of errors are represented in POBREP and the basic assumptions underlying Equations 1 and 3. We continue to examine the stencil printing operation. Consider the situation where the same quality characteristic, namely the solder paste volume, is measured at 20 different locations on a single part: five measurements are recorded per side. Figure 1 represents the pattern of deviations as a result of a squeegee pivot problem; outward arrows represent a positive deviation (excess volume), and inward arrows represent a negative deviation (insufficient volume).

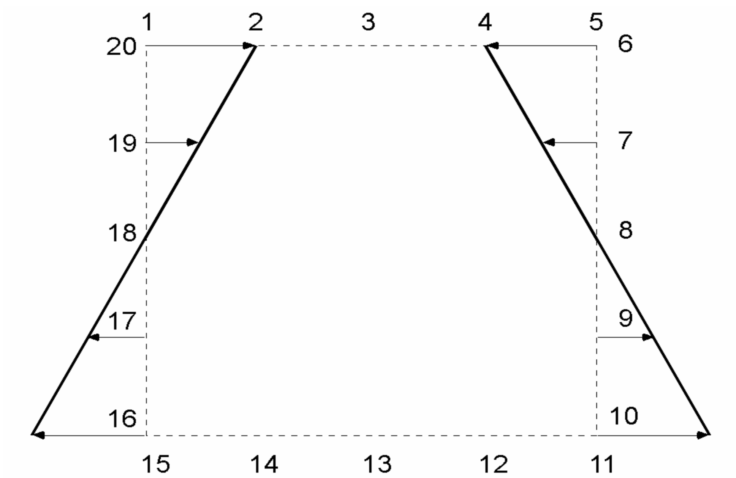


Figure 1 Deviation Pattern Corresponding to a Squeegee Pivot Problem in Stencil Printing

Figure 1 shows that the squeegee pivot problem causes deviations of solder paste volume only on the sides of the rectangular printing area; no deviations are expected on the top and bottom measurements. The thick lines connecting the arrowheads give a visual representation of the deviation pattern. The corresponding basis element, a, is defined as

$$a = (0, 0, 0, 0, 0, -1, -0.5, 0, 0.5, 1, 0, 0, 0, 0, 0, 1, 0.5, 0, -0.5, -1)', \quad (5)$$

where, $a_i$, the ith element of a, represents the error at position i in Figure 1. The advantage of this scaling scheme used in Equation 5 (using values scaled between –1 and 1) can be seen by considering a situation where the only error source that affects the process is the squeegee pivot problem, excluding all other error sources as well as common causes of variability. Then the resulting multivariate quality vector, x, will be a multiple of a: $x = z a$. Since the POBREP coefficient z will relate the component of x with the largest absolute value to a component 1 or -1 in a, it will give the maximum absolute deviation in actual measurement units. This provides directly interpretable information about the magnitude of the error; z is the maximum deviation from the nominal amount of solder paste at any of the locations (in fact, specifically at locations 6, 10, 16, 20). Further, in many processes as well as stencil printing there are practical limits to the maximum error that can occur at any location due to a particular quality problem. For instance, an excess of solder paste volume beyond a limit value is not possible given that the only error is a squeegee problem. This scaling scheme will be very useful in constructing the constrained solution space in the following section. In addition to ease of interpretation, scaling between –1 and 1 generates basis elements that are both generic and comparable with each other.

Since basis elements will be scaled from their original units and may differ substantially in terms of their original magnitudes, the scaling of each basis element should be performed separately. There are different ways to achieve the proposed scaling. Birgoren [18] compared them and proposed the following simple scheme: Let a be an m-dimensional basis element, and let $f(\cdot)$ be a scaling function from $R^m$ to $R^m$. Scale a basis element by multiplying it with a coefficient such that all components of the basis element lie between -1 and 1:

$$f(a) = k\cdot a,\ k\in R,\ a\in R^m. \qquad (6)$$

Here k is selected such that the maximum of the absolute values of the components of a is one, i.e. $\max_{1\leq j\leq m}(|a_j|) = 1$

The resulting POBREP coefficient will reflect the correct significance of the associated process problems. However, this scheme might theoretically yield basis elements with components of very low magnitudes: Let $a_1 = (10000, 1, 1, 1, 1, 1, 1, 1, 10000)'$ be a basis element in original measurement units, then the scaled element will be $f(a_1) = 0.0001\cdot a_1 = (1, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 1)'$.

It might be argued that a basis matrix with such basis elements will be ill-conditioned, hence problems of numerical instability might arise. Nevertheless, such basis elements are practically very unlikely, because basis elements are supposed to be constructed from process expertise, i.e. by the help of a process engineer, and a process engineer will ignore components of very low magnitudes and describe $a_1$, for instance, as $a_1 = (10000, 0, 0, 0, 0, 0, 0, 0, 10000)'$. In fact, this scheme brings certain advantages in terms of reducing the numerical errors. POBREP coefficients are computed by the following formula when the least squares method is used:

$$z = (A'A)^{-1}A'x. \qquad (7)$$

There are scaling techniques for reducing the numerical errors in solving a linear system $Ax = b$ [20]. Simple row scaling is one such technique. It transforms the linear system to $D^{-1}Ax = D^{-1}b$, where D is a diagonal matrix, such that each row in $D^{-1}A$ has approximately the same $l_\infty$ norm. Row scaling reduces the likelihood of adding a very small number to a very large number during elimination - an event that can greatly diminish accuracy. Row scaling is expected to reduce numerical errors for basic row operations.

Scaled basis elements, which are the columns of the scaled matrix A in Equation 7, have the same $l_\infty$ norm, which is 1. Since the components of A are all between -1 and 1, it can be argued that rows of A have approximately the same $l_\infty$ norm. Therefore the scaling scheme of Equation 6 is expected to reduce the numerical error when basic row operations are applied. However, it is also stated that there is no general scaling technique that guarantees reducing the numerical error [20].

There is a more important issue for the least squares method, regarding numerical stability; if the columns of A matrix in Equation 7 are nearly-dependent then least squares problems have sensitive solutions and sensitive minimum residuals with respect to perturbations in A and x [20]. Therefore non-orthogonality is a major concern for the numerical stability as well as a concern for the reliability of the POBREP coefficients. The scaling scheme does not bring any improvement in terms of orthogonality. We will discuss other ways of dealing with non-orthogonality in the following sections.

## 4. NON-ORTHOGONALITY AND CONSTRAINING SOLUTION SPACES

When the basis elements are orthogonal, POBREP establishes a reliable and accurate link between multivariate quality vectors and potential process errors characterized by a process-oriented basis. When the basis elements are not orthogonal different problems arise regarding the reliability and explanatory power of the POBREP coefficients. This section introduces the constrained solution spaces (CSS), showing the sort of problems for which it is intended. Also problems associated with non-orthogonality and linear dependency are described.

For many processes, there is a highest attainable level for the magnitude of each process error represented by a process-oriented basis element. How these limits arise is briefly discussed for the stencil printing example. The CSS technique proposed in this study uses these highest error levels to bound the POBREP coefficients from above or from below, or both; hence it imposes inequality constraints on the feasible space for the POBREP coefficients. Some of the problems associated with non-orthogonality are directly related to the common cause variation ε.in the underlying probability model of the POBREP methodology, $X = A z_0 + \varepsilon$.

If ε.did not exist, that is, $X = A z_0$, then $z_0$ would always fall inside the feasible space of $z_0$. Furthermore, if A is full rank, that is rank(A) = k where A is an m x k matrix and $k \leq m$, there would be a unique solution to $X = A z$, hence $z_0 = z$. On the other hand, common cause variation always exists, and it might contribute to a quality vector in such a way that the least squares solution to $X = A z + e$ produces a z vector that is outside the feasible space of $z_0$. Note that the least squares problem finds the POBREP coefficients even when A is a complete basis, that is, k = m; the only difference is that it produces a zero residual vector: e = 0. POBREP coefficients outside the feasible space of $z_0$ is considered unrealistically high in magnitude. The CSS technique avoids this by solving a constrained least squares problem. It gives a description of the type and the strength of underlying process problems that is less sensitive to changes in ε. for a non-orthogonal basis, and can reveal potential missing basis elements. When a quality vector is decomposed into known patterns using constraints on the magnitude of the coefficients, the contribution of each pattern will be calculated with respect to limits based on the physical process specifications, hence the strength of each problem will be constrained in a realistic way. Consequently, the residual vector e will contain the real amount process error that cannot be explained by the basis, and patterns in e will reveal potential missing basis elements.

Now, we proceed with a detailed discussion of consequences of non-orthogonality and linear dependency. We will first consider cases with a complete basis with k = m, and will extend these results to the cases with an incomplete basis with k < m. When the basis elements are not orthogonal three cases might arise:
(i) Process-oriented basis elements are slightly non-orthogonal and linearly independent,
(ii) Process-oriented basis elements are severely non-orthogonal and linearly independent, and
(iii) Process-oriented basis elements are linearly dependent.

These cases lead to different problems regarding the reliability of the POBREP coefficients. A certain amount of non-orthogonality can be tolerated, so the first case does not raise a serious concern; however, severe non-orthogonality causes unreliable POBREP coefficients [17]. This happens in two ways: First, small changes in the ε. Vector might result in highly different POBREP coefficients; hence POBREP solution becomes highly sensitive to the changes in the multivariate quality vector. Second, POBREP coefficients might be unusually large as compared to the magnitude of the components of the original multivariate quality vector x.

In this context, linear dependency can be regarded as an extreme case of severe non-orthogonality. When there is a dependency structure among the basis elements, there is not a unique solution for POBREP coefficients; instead, the solution is defined by an affine space. This situation can be thought of as a worst case of high sensitivity and unusually high coefficients.

There are well-established mathematical tools to measure the level of non-orthogonality among POB elements such as eigensystem analysis and singular value decomposition. Similarly, rank-deficiency of A will directly indicate that process-oriented basis elements are not linearly independent. Therefore, potential problems with POBREP can be identified prior to the application of methodology. This will also indicate the necessity of using the CSS technique in the POBREP methodology.

There is an equally important drawback of non-orthogonality that leads to another type of failure in the performance of the POBREP methodology: the POBREP coefficients might be unusually high in magnitude as compared to the magnitude of x. Figure 2 illustrates this problem using a simple example: It has two non-orthogonal basis elements, $a_1 = (0.75, 1)'$ and $a_2 = (1, 1)'$, and suppose that the multivariate quality vector x =

(1.125, 0.5)′ was generated as $x = 0.8a_2 + (0.325, -0.3)'$ where (0.325, -0.3)′ is a random perturbation due to common causes. This situation is illustrated as the actual mechanism in Figure 2. Using the POBREP basis this quality vector would be captured as $x = 3a_2 - 2.5a_1$. This is illustrated as the inferred mechanism in the figure.

POBREP methodology assumes that a quality vector is a multiple of a basis element if the only error source that is present is the one associated with the basis element. Therefore, one would be inclined to interpret the result $x = 3a_2 - 2.5a_1$ as follows: the problem associated with $a_2$ caused an error that is three times the magnitude of $a_2$. Similarly, the magnitude of the error associated with $a_1$ is two and a half times the magnitude of $a_1$.

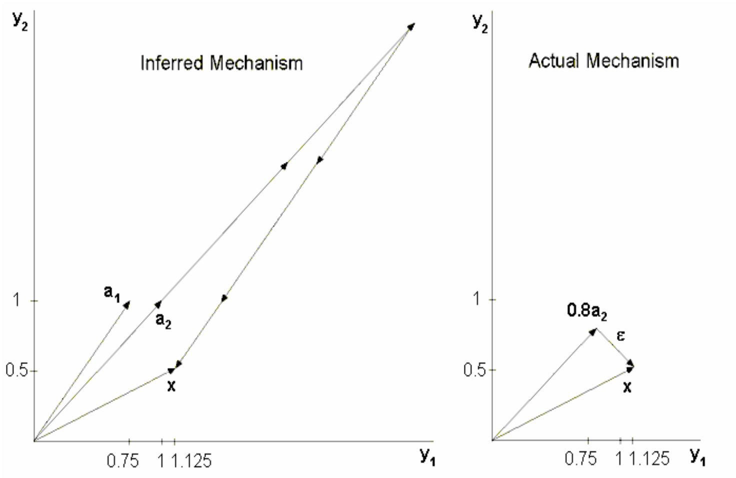


Figure 2. Non-orthogonal basis elements giving unusually high POBREP coefficients

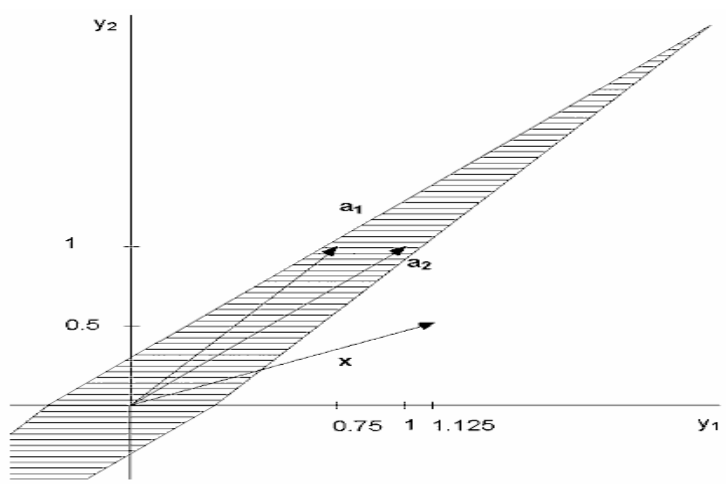


Figure 3. Feasible Space for the Multivariate Quality Vector for $-1.2 \le z_1 \le 1.2$ and $-1.2 \le z_2 \le 1.2$

Now suppose that the problems associated with $a_1$ and $a_2$ are considered to cause a serious quality problem when, $|z_1|$ and/or $|z_2| \ge 1.15$ and the practical bounds on $z_1$ and $z_2$ are such that $-1.2 \le z_1 \le 1.2$ and $-1.2 \le z_2 \le 1.2$. Then the coefficients $z_1 = -2.5$ and $z_2 = 3$ will be separately interpreted as a very significant quality problem, whereas, their combined effect is a quality vector whose largest component, $\|x\|_\infty = 1.125$ lower than the threshold for a significant problem. Worse yet, each coefficient is outside the practical limits for the magnitude of the respective problem. These apparent contradictions result from the non-orthogonality of the basis elements $a_1$ and $a_2$.

The common cause variation $\varepsilon$ plays an important role in the contradictions; its effect is illustrated in Figure 3. The shaded region in Figure 3 is the feasible region for multivariate quality vector that can be produced according to the constraints $-1.2 \leq z_1 \leq 1.2$ and $-1.2 \leq z_2 \leq 1.2$ when there is no $\tilde{\varepsilon}$ The multivariate quality vector x need not be in this region, since it is produced as a result of some POBREP coefficients within the lower and upper bounds plus some 'common cause' $\varepsilon$ vector. In fact, $x = (1.125, 0.5)' = 0.8a_2 + (0.325, -0.3)'$ of Figure 2 is not in the feasible region as shown in Figure 3. Note that the unrealistic POBREP solution $z = (-2.5, 3)'$ is a result of attempting to express x in terms of $a_1$ and $a_2$ in the presence of $\varepsilon$ without any regard to practical constraints on the physical magnitude of the POBREP coefficients.

One important point about the POBREP methodology that complicates this discussion is the case where the number of process-oriented basis elements is less than the dimension of the quality vector, that is, $k < m$. In this common situation, the least squares problem, $\min_z \|x - A z\|^2$, is solved to estimate the POBREP coefficients, and the solution is the projection of x onto the space spanned process-oriented basis elements. Then the previous arguments for x in this section holds for the projection of x. In fact, when the least squares method is applied in regression analysis, this situation is referred as multicollinearity, and it has received a significant attention in statistics literature because of the associated problems.

## 5. STRATEGIES FOR CONSTRAINING THE SOLUTION SPACE

In many processes it is possible to specify a highest level a process error can attain for each process problem; these levels can be imposed on the associated process-oriented basis components za as lower and upper bounds:

$$l \leq z\,a \leq u, \tag{8}$$

where l and u are m-dimensional lower and upper bound vectors. For the development of the bounds, the problem associated with a should be considered as the only cause of variation in the process; hence $x = z\,a$. The lower and upper bounds on z can be easily obtained if the basis elements are scaled using the scaling scheme described in Equation 6. The resulting POBREP coefficient will give the maximum absolute deviation in actual measurement units. Further, components in a reflect relative magnitudes of errors with respect to the largest component, which is either 1 or -1. Hence, the ith component of $z\,a$, $z\,a_i$, gives the magnitude of the error in the ith position of x in actual measurement units when $x = z\,a$. Therefore, it will suffice to specify bounds l and u in actual measurement units, which is an easy task for a process engineer.

The set of constraints in Equation 8 can be reduced to a single lower and upper bound for z, since a, l and u contain constant terms:

$$l \leq z \leq u. \tag{9}$$

There might be other ways of specifying the lower and upper bounds depending on the type of a process. For instance, a process might involve processing a three-dimensional object, and a coordinate measurement machine might measure different positions on the object for quality control purposes. Suppose that the measured errors can be organized in the following way: $x = (\Delta x_1, \Delta y_1, \Delta z_1, \Delta x_2, \Delta y_2, \Delta z_2, \ldots, \Delta x_m, \Delta y_m, \Delta z_m)'$, where, each group of $(\Delta x_i, \Delta y_i, \Delta z_i)'$ denote the measured deviations from the nominal in three dimensions at a position, hence, there are measurements at m different positions. In this case, the process-oriented basis element will be organized in the same fashion as the multivariate quality vector: $a = (a_{x_1}, a_{y_1}, a_{z_1}, a_{x_2}, a_{y_2}, a_{z_2}, \ldots, a_{x_m}, a_{y_m}, a_{z_m})'$. In this situation, it is more natural to define bounds based on geometric positions, rather than each component of a. If an upper bound can be defined for the magnitude of the three-dimensional error at a position, then the inequalities, $\| z\,(a_{x_i}, a_{y_i}, a_{z_i})' \| \leq u_i \; \forall i,\ 1 \leq i \leq m$, are obtained, which can be reduced to simple bound on z as in Equation 9.

Specific situations might require even more interesting sets of constraints, however, as long as the constraints are defined for individual process-oriented basis components, i.e. for a single $z\mathbf{a}$, the set of constraints will always reduce to a lower or an upper bound or both, since there is only one z involved in all of the constraints.

Constraining POBREP coefficients is not restricted to situations where highest attainable error levels can be specified. For certain process problems, the associated error pattern might occur always in one direction, where a direction can be defined according to the sign of the POBREP coefficient. That is, the process-oriented basis component for such problems will always have the form $0 \le z\mathbf{a}$, or $z\mathbf{a} \le 0$. These constraints can always be converted to a non-negativity constraint on z, by multiplying the basis element a by -1 when $z\mathbf{a} \le 0$: $0 \le z$.

**6. AN EXAMPLE**

It is helpful to revisit the example in Figure 2 to interpret a constrained least squares solution graphically. The actual mechanism shows the actual deviations caused by the two process problems and the common cause variation, and the resulting quality vector. The inferred mechanism in Figure 2, on the other hand, gives the unconstrained least squares solution, which includes no residual vector e; the solution contains unrealistically high POBREP coefficients. The constrained least squares problem for this example is as follows:

$$\begin{aligned} &\min \|(1.125, 0.5)' - z_1(0.75, 1)' - z_2(1, 1)'\|^2 \\ &\text{s.t.} \quad -1.2 \le z_1 \le 1.2, \text{ and} \\ &\qquad -1.2 \le z_2 \le 1.2\,. \end{aligned} \tag{10}$$

Since the quality vector x = (1.125, 0.5)′ is not in the feasible region as shown in Figure 3, the constrained problem finds the closest point in the feasible region to the quality vector x = (1.125, 0.5)′; this situation is illustrated in Figure 4. The optimal solution to the problem in Equation 10 is $z_c$ = (-0.484, 1.2)′ which yields the closest feasible point to x as $x_p$ = -0.484·(0.75, 1)′ + 1.2·(1, 1)′ = (0.837, 0.716)′. The residual vector e can be calculated by $e = x - x_p$ = (0.288, -0.216)′. The values of $z_0$, z, ε and e in these mechanisms are as follows:

| | |
|---|---|
| the actual values | : $z_0$ = (0, 0.8)′ and ε = (0.325, -0.3)′, |
| the unconstrained solution | : z = (-2.5, 3)′ and e = (0, 0)′, |
| the constrained solution | : z = (-0.484, 1.2)′ and e = (0.288, -0.216)′. |

Although neither of the z solutions from the unconstrained and constrained problem are close to the actual error vector $z_0$, the constrained solution is within the maximum error bounds and much closer to $z_0$ than the unconstrained solution.

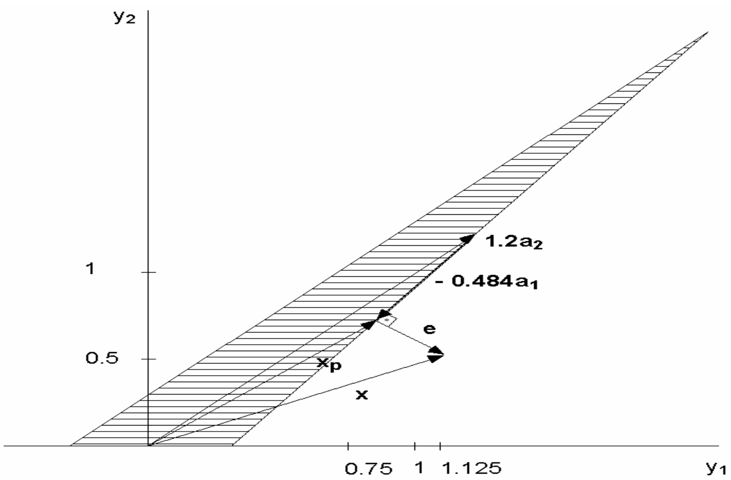


Figure 4. The Constrained Solution for the Example in Figure 3

## 7. CONCLUSIONS

This study presents an overview of the POBREP methodology, which aims to identify problem-related patterns in multivariate quality data by means of multivariate regression. This methodology gives misleading results if the problem-related patterns (process-oriented basis elements) are severely non-orthogonal, which is also known as multicollinearity problem in the regression analysis. Imposing practical bounds on the regression coefficients, which indicates the severity of the problem-related patterns in the methodology, provides a simple solution to multicollinearity. This study discusses first the scaling of process-oriented basis elements, a necessary condition for correct identification, then how multicollinearity arises in POBREP. The implementation of the bounds in production processes are examined, and illustrated on an example.

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