Technology, 12(3), 157–165, (2009)

TECHNOLOGY

LOAD CAPACITY OF A PIVOTED-PAD THRUST BEARING USING SIMULATED ANNEALING

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Abstract

This study gives a detailed mathematical model of a single pivoted-pad in order to obtain maximum load carrying capacity and the pressure take place in the pivoted-pad thrust bearing using the Simulated Annealing (SA) optimization technique. The SA is based on the idea of exploring the solution space by moving around in the neighborhood structure for the global optimum point. It does not require the evaluation of gradient of the objective function. It imitates the process of annealing in metals as they cool from liquid to solid states. Results checked against the numerical solution to demonstrate capability of the technique which produces efficient solutions.

Key Words: Pivoted-pad, thrust bearing, simulated annealing, optimization

1. Introduction

Modern high speed machineries are complex. With increasing performance criteria, the design process of these system usually require the integration of the design and analysis by ensuring the safe and reliable operation of pivoted-pad thrust bearings, one of most important element in such machinery. Pivoted-pad thrust bearings are customarily used to support high speed machinery. They eliminate oil whirl instability and other destabilizing factors, which contribute to system instability, such as thermal instability, internal friction, and cross-coupling [1]. In this type of bearing, the pads are free to follow the rotor (runner) and the pad tilts to compensate for misalignment between the pad and the rotor. The forces produced in the bearing are not capable of driving the rotor in unstable mode. Each pad is pivoted at a point to create a converging fluid film. Interested readers can refer to the studies [2], [3], and [4]. As the complexity of the system is increased, the need to develop a computational environment that allows a natural exchange of data and results between producers leading to integrated approach to optimal design has become evident. Traditional optimization methods are highly sensitive to starting points and easily converge to local optimum not to global optimum solution [5].

Developments in computer technology have proved to be a great chance to the world of design optimization. Many non-traditional optimization methods such as Genetic Algorithms and Particle Swarm Algorithms have been utilized to solve mechanical design optimization problems, see references [6] and [7]. Any efficient optimization algorithm explores to investigate new and unknown areas in search space and exploit to make use of knowledge found at point previously visited to help find better solution point. From this point of view, Simulated Annealing (SA) is utilized for a pivoted-pad thrust bearing design. The SA is a powerful optimization technique and it has the ability to find global optimum for non-linear problems. The SA can provide a remarkable balance between exploration and exploitation of the search space. From this point of view, this study provides use of the SA to seek a global optimum solution to problem in hand. The SA algorithm imitates the process of annealing in metals as they cool from liquid to solid states. The algorithm is based on the idea of exploring the solution space by moving around in the neighborhood structure for the global optimum point. It does not require the evaluation of gradient of the objective function.

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2. The Simulated Annealing

In this section of the paper, the fundamental intuition of the SA and how it processes are given briefly. The SA was proposed by Kirkpatrick et al. [8] to deal with complex non-linear combinatorial optimization problems. They showed the analogy between simulating the annealing of solid as proposed by Metropolis et al. [9]. The SA is an iterative improvement algorithm for a global optimization. It is inspired from thermodynamic to simulates the physical process of annealing [10] and [11] of molten metals. It obtains the minimum value of energy by simulating annealing which is a process employed to obtain a perfect crystal by gradual cooling of molten metals [12] in order to keep the system of melt in a thermodynamic equilibrium at given temperature. Thus, it exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure. At high temperature, the atoms in the molten metal can move freely with respect to each other as the cooling proceeds, the atoms of metal become more ordered and the system naturally converges towards a state of minimal energy. This formation of crystal mostly depends on the cooling rate. If the metal is cooled at very fast rate, the atoms will form an irregular structure and the crystalline state may not be achieved. The Metropolis algorithm provides an efficient simulation of a collection of atoms in equilibrium at given temperature. The SA makes use of the Metropolis et al. [9] algorithm which provides an efficient simulation according to a probabilistic criterion stated as:

$$P(\Delta E) = \begin{cases} 1, & \text{if} \quad \Delta E \langle 0 \\ e^{(-\Delta E/T)}, & \text{otherwise} \end{cases} \tag{1}$$

Thus, if $\Delta E \langle 0$, the probability, P, is one and the change - the new point- is accepted. Otherwise, the modification is accepted at some finite probability. Each set of points of all atoms of a system is scaled by its Boltzmann probability factor $e^{(-\Delta E/T k)}$ where ΔE is the change in the energy value from one point to the next, k is the Boltzmann's constant and T is the current temperature as a control parameter. Even at a low temperature, there is a chance for the system being in a high-energy state. Thus, there is a corresponding chance for getting out of a local energy minimum in favor of a better solution, a global one. The general procedure for employing the SA as follows;

Step 1: Start with a random initial solution, X, and an initial temperature, T, which should be high enough to allow all candidate solutions to be accepted and evaluate the objective function. The initial temperature is problem specific and depends on the scaling of the objective function.

Step 2: Set $i = i + 1$ and generate new solution ($X_i^{new} = X_i + r \ \ SL_i$) where r is random number and SL_i at each move should be decreased with the reduction of temperature. Evaluate $F_i^{new} = F(X_i^{new})$.

Step 3: Choose accept or reject the move. The probability of acceptance (depending on the current temperature) if $F_i^{new} \langle F_{i-1}$, go to Step 5, else accept F_i as the new solution with probability $e^{(-\Delta E/T)}$, where $\Delta E = F_i^{new} - F_{i-1}$ and go to Step 4.

Step 4: If F_i was rejected in Step 3, set $F_i^{new} - F_{i-1}$. Go to Step 5.

Step 5: If satisfied with the current objective function value, F_i, stop. Otherwise, adjust the temperature ($T^{new} = T \ r_T$) where r_T is temperature reduction rate called cooling schedule and go to Step 2. The process is done until freezing point is reached. The freezing point is the lowest energy state possible where the atoms are a pattern that corresponds to the global energy minimum of a perfect crystal. The major advantages of the SA are an ability to avoid becoming trapped in local optimum. This is due to nature ability of the SA allowing deteriorations with a large probability in the objective function.

3. Formulation of the Problem

The pivoted-pad bearing was invented by Anthony G.M. Mitchell in 1905 and independently by Albert Kingsbury in 1910 in a slightly different type [13]. Nowadays, the pivoted-pad thrust bearing (see Figure1) is designed to transfer high loads in applications due to good operational lifespan and minimum power loss. In order to attain hydrodynamic lubrication, the pivoted-pad should be placed at an angle to the oil flow so that a converging wedge is produced as shown in Figure 2.

Figure 1 A pivoted-pad thrust bearing [14].

The mathematical modeling of fluid film lubrication separating pad and runner in the pivoted-pad thrust bearing consists of a flat surface sliding over a pivoted-pad is shown in Figure 2. The pad is stationary while the runner is moving at a driving speed, U. Due to the direction of the driving velocity, the fluid is pulled into the bearing and proceeds through a converging wedge resulting with pressure generation. The generated pressure will force fluid out both the leading and trailing edges, and provides a force to carry a load.

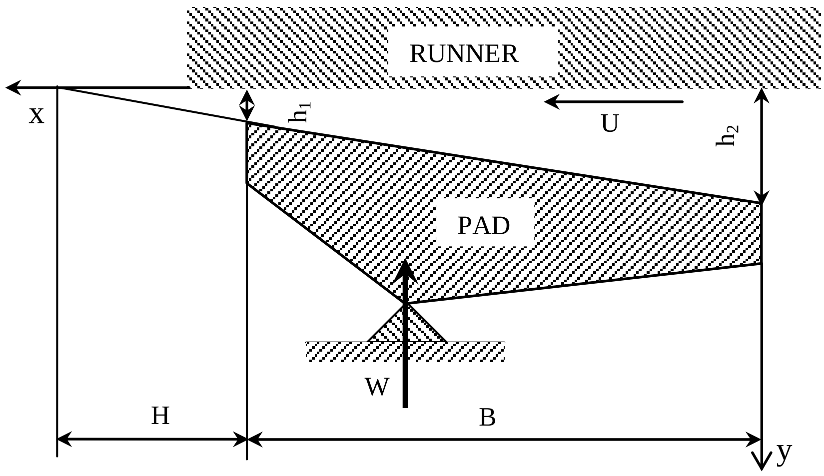

Figure 2. Schematic representation of a single pivoted-pad thrust bearing

The initial phase of utilizing an optimization technique to the pivoted-pad needs modeling of the system behavior. Mathematical expression of load carrying capacity for a single pivoted-pad and the derivation of governing the expression are presented following an approach found in [15]. The model is based on the Reynolds equation for pressure between pad and runner. Pressure, p, is a function of x. Reynolds equation in one dimension is given as:

$$dp/dx = -6U\eta[(h - \bar{h})/h^3] \quad (2)$$

$$h = (xh/H) \quad (3)$$

$$H = B[h_1/(h_2 - h_1)] = B/K \quad (4)$$

$$K = [(h_2 - h_1)/h_1] \quad (5)$$

The pad convergence ratio, K, is a function of position of the pivot only. It is independent of the operating conditions. The load carrying capacity depends strongly on convergence ratio.

$$d_x = H(dh/h_1) = B[h_1/(h_2 - h_1)\, dh/h_1 = (B/K)\, dh/h_1 \tag{6}$$

$$dp = -6U\eta[(dx/h^2) - (\bar{h}dx/h^3)] = -6U\eta(B/Kh_1)[(dh/h^2) - (\bar{h}dh/h^3)] \tag{7}$$

$$p = -6U\eta(B/Kh_1)[-(1/h) + (\bar{h}/2h^2) + c] \tag{8}$$

where p is pressure, U is surface velocity in x direction, B is length in direction of motion, η is dynamic viscosity, c is the constant of integration, $\bar{h} = h$ is at peak pressure, h_1 is the fluid film thickness at the leading edge, h_2 is the fluid film thickness at the trailing edge, and L is length (for a section of infinite pad).

The boundary conditions are $h = h_1$ and $h = h_2$, $p = 0$ at either end of pad. The formulation of governing equation begins with the boundary conditions established.

$$-1/h_1 + \bar{h}/2h_1^2 + c = 0 \tag{9}$$

$$-1/h_2 + \bar{h}/2h_2^2 + c = 0 \tag{10}$$

$$(\bar{h}/2)[(1/h_1^2) - (1/h_2^2)] + (1/h_2) - (1/h_1) = 0 \tag{11}$$

$$\bar{h} = [(h_2 - h_1)/(h_2\, h_1)][2h_1^2 h_2^2/(h_2^2 - h_1^2)] = 2h_1h_2/(h_2 + h_1) \tag{12}$$

$$c = (1/h_1) - (\bar{h}/2h_1^2) = (1/h_1) - 2h_1h_2/(2h_1^2(h_2 + h_1)) = 1/(h_2 + h_1) \tag{13}$$

Substituting Equation (13) into Equation (8) becomes

$$p = -6U\eta(B/Kh_1)[-(1/h) + (2h_1h_2/(h_2 + h_1))(1/2h^2) + (1/(h_2 + h_1)] \tag{14}$$

$$p = 6U\eta(B/Kh_1)[(1/h) - (h_1h_2/(h_2 + h_1)h^2) - (1/(h_2 + h_1)] \tag{15}$$

The load carried per unit length W/L is the integral of the pressure over the pad

$$(W/L) = \int_{H}^{H+B} p\,dx = [B/(h_1K)] \int_{h_1}^{h_2} p\,dh \tag{16}$$

where W is supporting load. Substituting p into Equation (16) becomes

$$(W/L) = (B/(h_1K))\, 6U\eta(B/(Kh_1)) \int_{h_1}^{h_2} [(1/h) - [h_1h_2/(h_2 + h_1)h^2] - 1/(h_2 + h_1)]dh \tag{17}$$

Rearranging and integrating

$$(Wh_1^2/6U\eta LB^2) = (1/K^2)[(\ln h) + (h_1h_2/(h_2 + h_1)h) - h/(h_2 + h_1)]\Big|_{h_1}^{h_2} \tag{18}$$

$$(Wh_1^2/6U\eta LB^2) = (1/K^2)[\ln(h_2/h_1) + ((h_1h_2/(h_2 + h_1))((1/h_2) - (1/h_1))) - ((h_2 - h_1)/(h_2 + h_1))] \tag{19}$$

$$(Wh_1^2/6U\eta LB^2) = (1/K^2)[\ln(h_2/h_1) - (2(h_2 - h_1)/(h_2 + h_1))] \tag{20}$$

$$W = (6U\eta LB^2/h_1^2)(1/((h_2/h_1) - 1)^2)[\ln(h_2/h_1) - (2(h_2 - h_1)/(h_2 + h_1))] \tag{21}$$

$$W = (6U\eta LB^2/h_1^2)(1/(\lambda-1)^2)[\ln\lambda - (2(\lambda-1)/(\lambda+1))] \quad (22)$$

$$W = (U\eta LB^2/h_1^2)F_w \quad (23)$$

$$F_w = (6/(\lambda-1)^2)[\ln\lambda - (2(\lambda-1)/(\lambda+1))] \quad (24)$$

where $\lambda = h_2/h_1$ and F_w is load factor

4. Employing the Simulated Annealing

The statement of the problem is formulated to find the maximum load carrying capacity and pressure for the given bearing parameters with respect to design variables h_1 and h_2 as follows:

$$W(h_1, h_2) = (6U\eta LB^2/h_1^2)(1/((h_2/h_1)-1)^2)[\ln(h_2/h_1) - (2(h_2-h_1)/(h_2+h_1))] \quad (25)$$

$$p(h_1, h_2) = 6U\eta(B/Kh_1)[(1/h) - (h_1h_2/(h_2+h_1)h^2) - (1/(h_2+h_1))] \quad (26)$$

Subject to

$$0.0010 \le h_1 \le 0.0025 \quad (27)$$

$$0.0 \le h_2 \le 0.0010 \quad (28)$$

Parameters used are oil dynamic viscosity, $\eta = 2x10^{-6}$ reyn (0.01378 kg/m–sn), surface velocity, $U = 50$ ft/sec (15.24 m/sn), length in the direction of motion, $B = 5.0$ inch (127 mm), and length for a section of infinite pad, $L = 0.50$ inch (12.7 mm).

By employing the SA, a random initial point is selected at high temperature and a series of moves are made according to defined annealing schedule. The annealing schedule determines the degree of uphill movement permitted during the search. The change in the objective function values, ΔE, is computed at each move. A new solution is generated in the neighborhood of the current configuration in each iteration. This new solution is automatically accepted with probability of 1, if it results in decreased objective function value. Otherwise, if the new solution is increased the objective function value, the acceptance is given with a small probability, $e^{(-\Delta E/Tk)}$. Where T is the current temperature and k is Boltzmann's constant. The probability expression suggests that if the temperature of the system is large, the probability of accepting the solution increases. Otherwise, if T is low, the probability of accepting solution decreases. Therefore, the temperature needs to be high at the beginning. As the iteration proceeds, the temperature is gradually decreased until the stopping condition is met. There are many ways to determine when to stop running the algorithm: One is the temperature when reduced to a threshold. Another is to reach a pre-specified number of temperature transitions. All the generating and acceptance depend on the temperature. The global optimum can be converged by carefully controlling the rate of cooling of the temperature. The important setting parameters of the SA for this study are chosen as follows: Initial temperature $T = 100000$, temperature reduction rate $r_T = 0.5$, and number of iterations performed at a particular temperature $n = 5$.

5. Result and Discussion

In Figure 2, it can be seen that the fluid film lubrication in the pivoted-pad thrust bearing generates pressures in a viscous fluid dragged into a converging wedge existing between the surfaces within h_1 (the fluid film thickness at the leading edge) and h_2 (the fluid film thickness at the trailing edge) in a relative motion. In Figure 3, it should be noted from the SA results that the maximum pressure is $4.28x10^{-2}$ psi (30.09 kg/m^2)

at $h_1 = 1.93x10^{-3}$ inch (0.0490 mm) and $h_2 = 8.00x10^{-4}$ inch (0.02032 mm). The variation of load capacity factor with the pivoted-pad inclination is shown in Figure 4. The systematic evaluation of how load capacity factor varies with inclination of the bearing using analytical solution is given. The distribution of function values by the SA during search with converging on global optimum point is also given on the same plot. There is a very good agreement between the calculated and the SA results. It can be noted that the maximum load capacity factor, 0.160, is obtained with the inclination ratio, h_1/h_2 , value of 2.19. Also Figure 5 gives variation of load capacity factor versus h_1 and h_2. Global optimum point for the load capacity factor 0.160 at $h_1 = 1.612x10^{-3}$ inch (0.0409 mm) and $h_2 = 7.351x10^{-4}$ inch (0.0186 mm) by the SA is given on the same plot. Figure 6 gives plot of the load supported by a single pivoted-pad versus h_1 and h_2. The maximum value of the load supported found via the SA after 38773 function evaluations (upon termination) is 6168.71 pound (2798.08 kg) which is the same as numerical solution at $h_1 = 1.0x10^{-3}$ inch (0.0254 mm) and $h_2 = 9.99x10^{-5}$ inch (0.0025 mm).

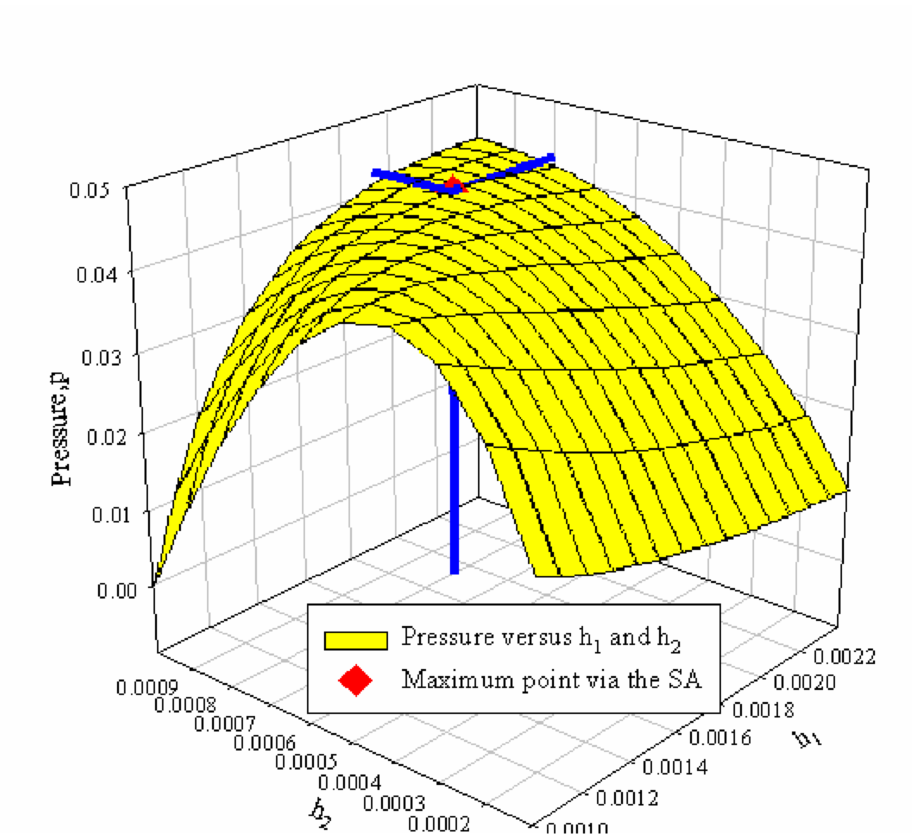

Figure 3. Lubrication pressure profile versus h_1 and h_2

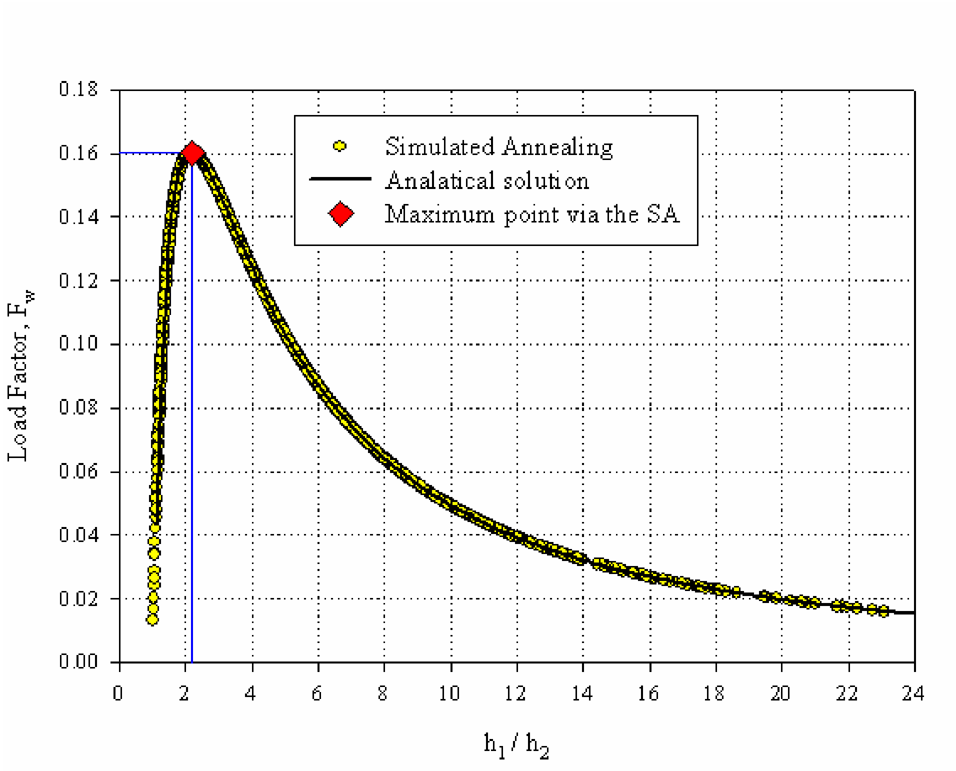

Figure 4. Variation of load factor versus pad inclination, h_1 / h_2

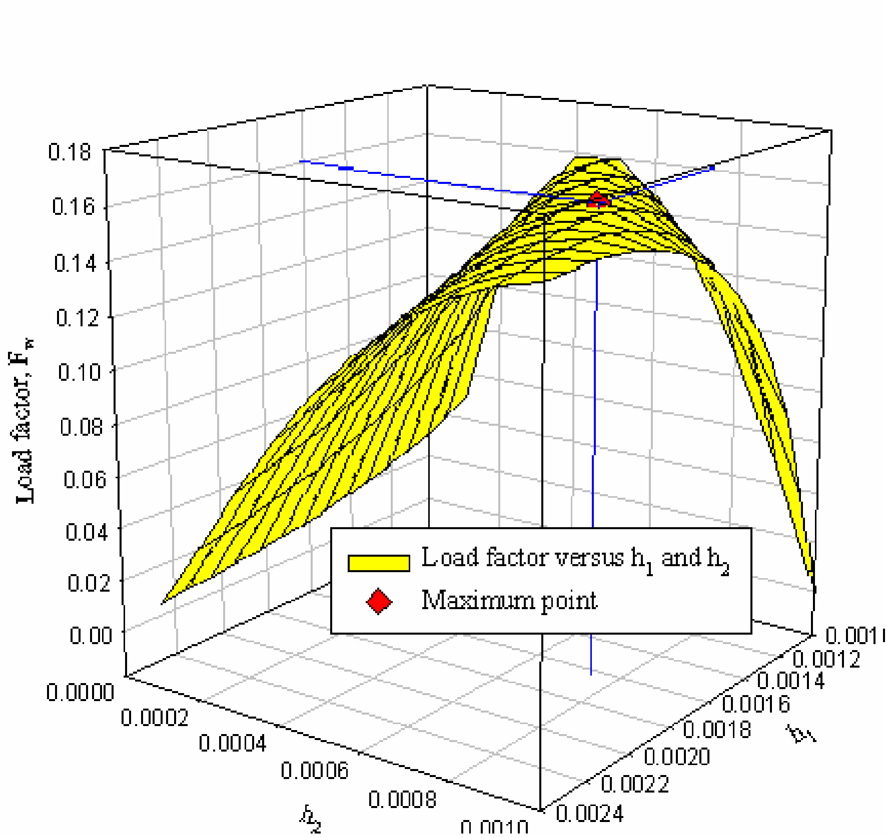

Figure 5. Variation of load factor versus h_1 and h_2

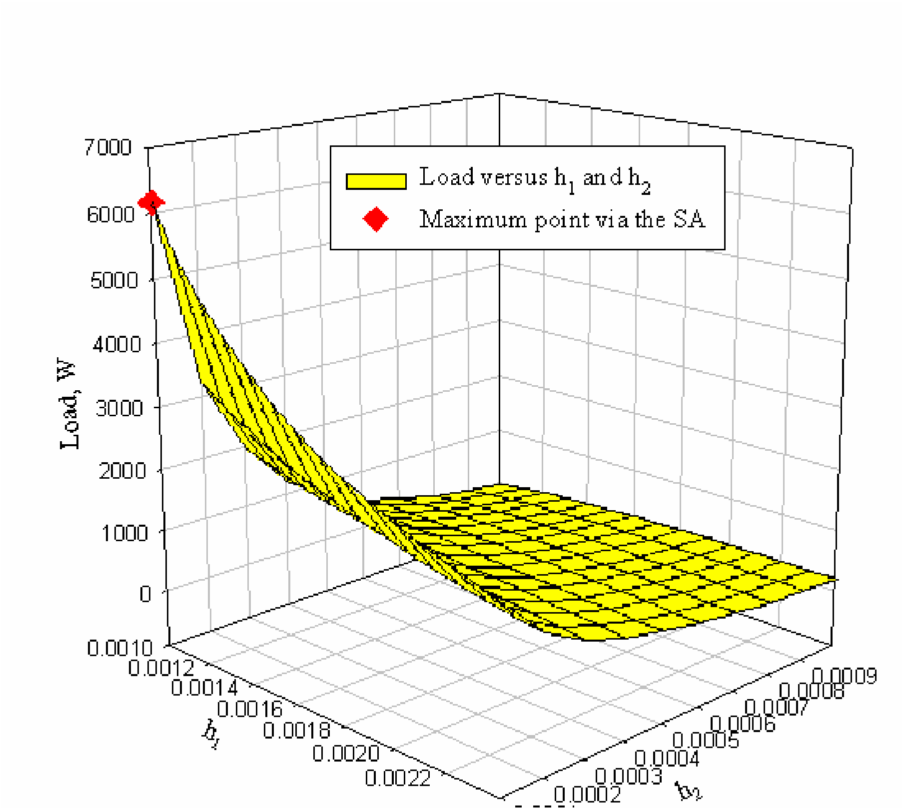

Figure 6. Load capacity versus h_1 and h_2

6. Conclusion

This study employs a nature inspired algorithm called simulated annealing (SA) to find the maximum load carrying capacity and pressure distribution for a single pad of a pivoted-pad thrust bearing. The load components are obtained by integrating the pressure in the film converging using Reynolds equation. A numerical study is also conducted to show the efficiency and applicability of the SA for the problem in hand. Results checked against the numerical solution to demonstrate capability of the technique used. It is worth trying to compare the results obtained by numerical solution. The SA offers a guarantee of optimum point and finds the global optimum with a high probability as in this study. The major disadvantage of the SA is computation intensive. However, this is not significant with current advances in computers and computing techniques capability. It can be concluded that nature inspired algorithm is proven to be robust and has demonstrated its capability to produce an efficient solution.

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