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COMPARATIVE ANALYSIS OF THE PRESSURE VARIATIONS AND THE FLOW RATES OF A HYDRAULIC PUMP MADE OF A PAIR OF ELLIPTICAL AND CYLINDRICAL SPUR GEARS

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Abstract

In this study, the pressure variations and the flow rates of an elliptic spur gear hydraulic pump with 2 modules and 22 teeth between 50 rev/min and 200 rev/min have been realized. The results were compared with those obtained analytically from a cylindrical spur gear hydraulic pump with 2 modules and 22 teeth. In addition, comparing the empirical results of spur gear hydraulic pump analytically analyzed and those of elliptical gear hydraulic pump, it has been observed that the elliptical gear hydraulic pump has more efficiency and as the cycle increases, so does the difference of flow rates between the two types of pumps. The experiments with the elliptical gear hydraulic pump produced were done at slow circles. The reason for this is that there are still some experiments in medical world to test the usage of elliptical gear hydraulic pumps in artificial heart and dialysis machines.

Key Words: Non-Circular Gears, Elliptical Spur Gear, Hydraulic Pump

1. Introduction

It has been discovered that the first samples of non-circular gears were used by Leonardo da Vinci in the 17th century in clocks, musical instruments, automatic play tools, key making machines, Geneva mechanisms and pumps. [1, 2, 3].

Non-circular gears are simply used in multi-dimensional mechanics. [4-6]. Elliptical gears are successfully used in the paper bag machines [7], the flow rate meters, the typewriters, the parceling machines, the labeling machines, the textile machines and the cigarette machines [8]. Other applications of them are observed to be in non-circular potentiometers for the control of electro-mechanical systems, in the transfer, the design and the measurement of continuously changing movement and force with low friction, in the direct current meters, in the robots and in the calculators [9-14]. In addition, these mechanisms have also been used in the oscillators with various frequencies, in the ground service equipments of space shuttles and in the shuttles themselves [15-18].

In literature, some computer programs have been developed to draw the teeth profiles of rack cutters and elliptical spur gears. It has been discovered that the under-cutting lines of elliptical spur gears produced by these programmes developed have been examined, and that the teeth profiles of gears produced by Rolling method and those of the elliptical gears cut on wire EDM have been compared [19]. Some studies concerning the production of elliptical gears comprise of the theoretical models that take into account one cutter that only makes Rolling movement on the section circle [20]. On the other hand, the developments in the computer-based designs and in machine tool technology have made the production of elliptical gears more economical and more efficient [21-23]. Researches on non-circular gears have, in recent years, focused on basic mathematical analysis, designs, production and application areas [24-26]. Some general design equations have been prepared that could be applied to all types of different non-circular gears [27, 28].

In this study related to the comparison of the mathematical results of spur gear hydraulic pump and empirical results of elliptical gear hydraulic pump, it has been observed that the elliptical gear hydraulic pump has

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more efficiency and as the cycle increases, so does the difference of flow rates between the two types of pumps. The experiments with the elliptical gear hydraulic pump produced were done at slow circles.

2. Design of Elliptical Spur Gears

The curve circling the surfaces obtained by cutting inclined to the bottom of cylindrical and conical is called an ellipse. The points that create the distance from the two focal points is fixed and equal to the bigger axis. The geometric positions of these points are called elliptical coordinator. Polar equations of elliptical curves are calculated with the following equations:

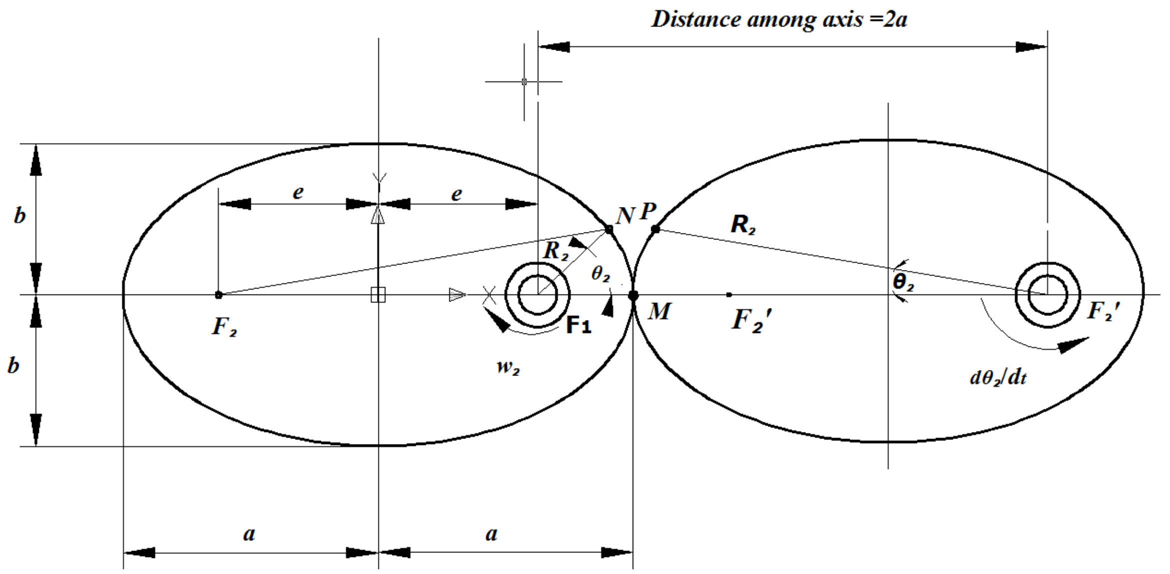

Figure.1. A pair of ellipse

Polar equation of elliptical curves is;

$$R_k = \frac{P_k}{1 - e_k \cos(k\phi)} \quad \text{Pk, ek} \in \text{R, k=1,2,….,r} \tag{1}$$

In this equation R, the set of real numbers (in case k=1 the pitch curve is a classical ellipse and the rotation shaft coincides with one its foci). In elliptical curve equation eccentricity of the ellipse is obtained by “ek” Eq.2.

$$e_k = \frac{\sqrt{a^2 - b^2}}{a} \tag{2}$$

In elliptical curve equation “P” is obtained with Eq.3.

$$P = a(1 - e_k{}^2) \tag{3}$$

The curve Rk has $k_{R_{k\max}}$ and $k_{R_{k\min}}$ values. The pitch curve has 2k congruent part between $k_{R_{k\max}}$ and $k_{R_{k\min}}$ values, that can be transformed to each other by rotation and/or mirroring.

A module programme has firstly been developed to obtain elliptical curves by preparing AutoLISP programme that calculates and draws the polar equation of elliptical gears.

In this second phase of study gear production by rolling method and rack cutter were taken into account. Since other methods of gear production are not dealt with, only the data of rack cutters on CAD were used.

Teeth surfaces of rack cutters are composed of two angular and symmetrical lines that constitutes a (α) angle according to the main axis of 2-D coordinate system. Complex numbers were used to mathematically obtain the coordinates of the cutter in 2-D coordinate system and the following equations were obtained:

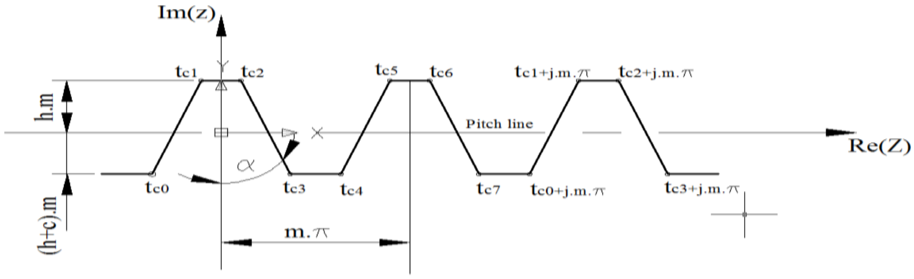

Figure 2. 2-D model of cutter profile.

Cutter coordination points were calculated with following equations;

$$\left.\begin{aligned} t_{c0} &= m\left(h_1 - i\left(\frac{\pi}{4} + h_1.\tan(\alpha)\right)\right) \\ t_{c1} &= m\left(-h_2 - i\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right) \\ t_{c2} &= m\left(-h_2 + i\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right) \\ t_{c3} &= m\left(h_2 + i\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right) \end{aligned}\right\} \quad [16] \tag{4}$$

where i2 = -1, see Fig. 3. The pitch line of the cutter is coinciding with the complex axe. The fillet radiuses of standard tool are in this case neglected. Complex co-ordinates of sth corner point in the rack

$$t_s = t_{s-4} + I.m.\pi \tag{5}$$

S= 4,5,……, 4Z-1 where Z = Z+1, is the total number of teeth of basic rack.
A different module programme has been developed to obtain required number of teeth profile on AutoLISP using these equations. The sample rack cutter produced by module programme is given in Fig.3 a.

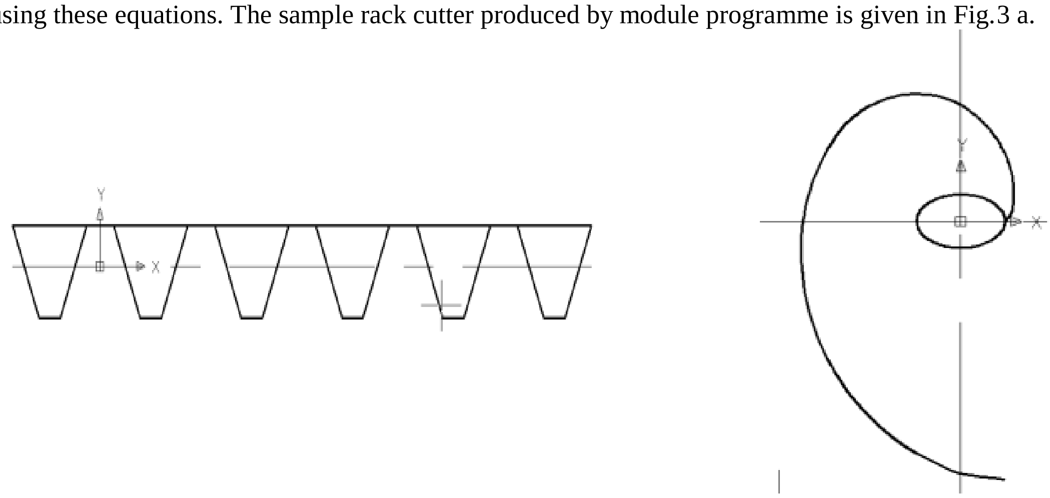

Figure 3. a) The obtained cutter profile on AutoCAD, b) The obtained the involute of 2 lobe ellipse on AutoCAD

It is necessary to obtain involute of the ellipse to design elliptical gears on AutoCAD with Rolling method. Involute of the ellipse was obtained by the following equations:

The arc length of the ellipse is;

$$s_t = \int_0^t \sqrt{a^2 \sin(x)^2 + b^2 \cos(x)^2}\, dx \tag{6}$$

and the involute of the ellipse is;

$$x(t) = a\cos + \frac{a \sin_t \sin(t)}{\sqrt{a^2 \sin(t)^2 + b^2 \cos(t)^2}} \tag{7}$$

$$y(t) = b\sin(t) - \frac{b s_t \cos(t)}{\sqrt{a^2 \sin(t)^2 + b^2 \cos(t)^2}} \tag{8}$$

Involute of the ellipse was obtained by using these equations through AutoLISP on AutoCAD (Fig.3.b)

The basic involute profile curves of gears can be defined by the normal profile numbers of rack cutters. Their normal lines coincide with tangential lines on contact point (Fig. 4).

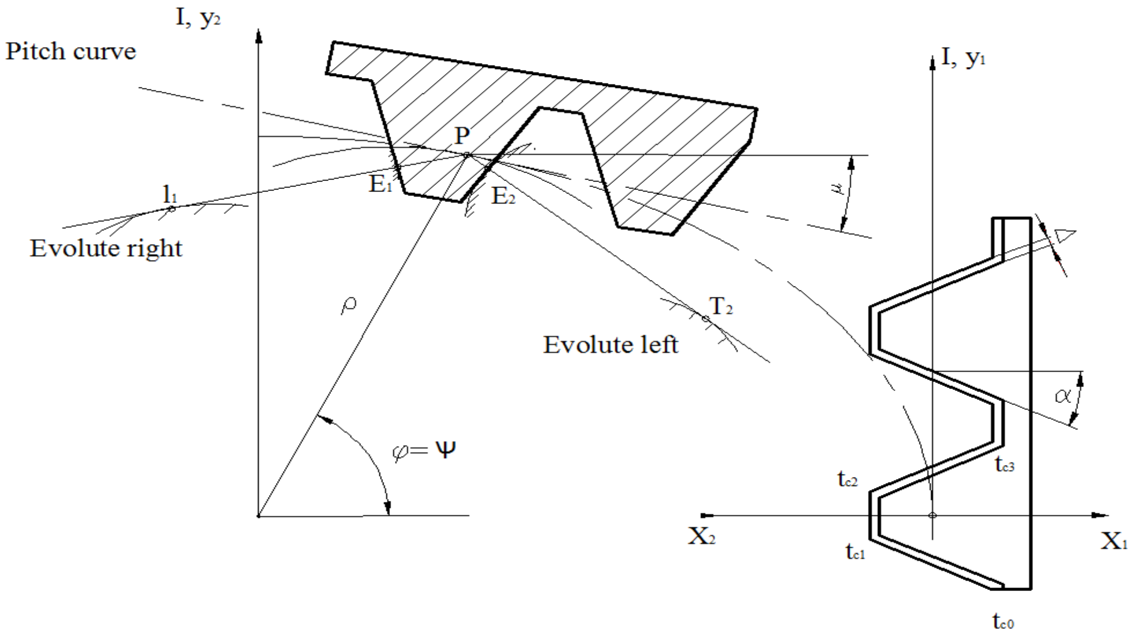

Fig. 4. The ellipse of the section where cutter is rolled

After obtaining the teeth profile of elliptical gears at 900 arc line with this method, elliptical gear can be produced by using move, rotate, subtract and mirror commands. One of the reasons for making the design as 2-D on AutoCAD is to make use of these standard commands. On the other hand it has been though to use practically the teeth data to produce gears with wire EDM. Δ difference shown in Fig.4 comes from the process itself as wire diameter and spark distance.

Pitch curve rolls without sliding on elliptical pitch curve. Where angular position of the pitch curve is φ=ψ, the pitch line of the cutter is in fact the tangent of the pitch curve. The complex coordinates at Sth corner points of the teeth of the pitch cutter:

$$w = (t_s - \mathrm{I}\int_0^{\psi} \sqrt{\rho^2 + \left(\frac{\partial}{\partial \phi}\rho\right)^2} d\phi))e^{(I\mu)} + \rho e^{(I\psi)} \quad [16] \tag{9}$$

and curve angle of tangential lines is;

$$\mu = \psi + \frac{\pi}{2} + \theta \quad [16] \tag{10}$$

The angle between diameter vector and tangential line can be obtained by:

$$\theta = \arctan\left(\frac{\rho}{\frac{\partial}{\partial \phi}\rho} \right) \quad [16] \qquad (11)$$

Module calculation of the teeth can be obtained by;

$$m = \frac{\int_0^{\frac{\pi}{k}} \sqrt{R^2 + \left(\frac{\partial}{\partial \phi} R \right)^2} \, d\phi}{\xi \pi} \qquad (12)$$

The complex coordinates of one tooth's corner points that shape the cutter were calculated by taking Eq.9. into account.

A module programme has been developed that realizes 2-D design of elliptical gears at required teeth number and modules on AutoCAD by transferring Eq.9 - Eq.12 on AutoLISP.

The 2-D image of the elliptical gear was prepared on AutoCAD according to the rolling method with AutoLISP programme where a=15,2 mm, b=27,03 mm, Z=22 and m=2

3. Manufacturing of Elliptical Spur Gears

Elliptical gears, which are very difficult to be produced by traditional methods and require manufacture lines equipped with special techniques are important members of non-circular gears group. They are widely preferred in flow measurement tools and in the production of the pumps with high flow rate. In this study elliptical spur gears were produced by using rotation, copy, measurement and macro programming APT functions at CNC wire EDM.

The cutting codes of the gears designed by AutoCAD were produced by MasterCAM. One pair of elliptical gear was produced from aluminium by installing the codes produced on wire EDM.

4. Design, Production and Empirical Data of The Pump Elliptical Spur Gears

Hydraulic pumps are used at various structures and features depending on the requirements of the hydraulic systems. It is known that pumps are widely used for transferring hydraulic power. Elliptical gear hydraulic pumps with high pressure can also be used as hydraulic motors due to their cost-efficiency and high performance.

In gear pumps, wide cross sectional areas in pump stem lead to high pressure, and thus high pressure causes big axis and radial forces. The design of the pump stem with optimum geometry required to stand these forces is very significant with regard to the choice of optimum gear, to economic use of materials and to the maximum efficiency of the pump. The fact that these types of pumps have positive transmission causes gears and pump stem to be affected by different hydraulic pressures and dependent forces at the same time. It is accepted that there is no fluid leakage at the moment of the contact of the twin teeth of the driver and driven gears when the pump is running.

In gear pumps, the pressure at the absorption line is less than atmospheric pressure, and system pressure at the exit line. This pressure difference causes pump stem, bearings and gears to be affected by big axis and radial forces. The magnitudes of these forces change depending on the gear geometry, gear dimension, flow rate and work pressure.

The fact that spur gear pumps and motors have been widely used in industrial applications and that elliptical spur gears have gained priority in design and manufacture of flow rate meters have all led to the examination of the main characteristics of a pump comprising of a pair of elliptical spur gears in this section.

A pair of elliptical spur gears was produced on CNC wire EDM with 2 modules, 22 teeth and 10 mm tooth width (Fig.5)

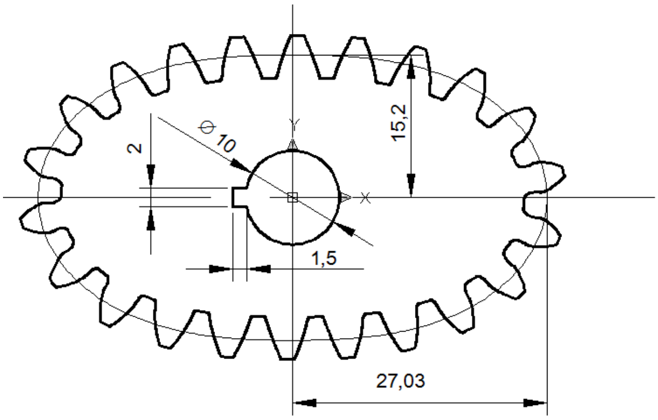

Figure 5. Elliptical spur gear used in the pump produced

The stem design was made with the help of Solidworks in compliance with the gears produced (Fig.6). The working clearance between two axis was given c+0,01 mm for elliptical spur gears in pump designs. In addition, The working clearance between two axis is suggested to be 0,2xm for gear pumps in literature. Aluminium blocks were produced on CNC milling machine according to the measurements of the pump produced (Fig.7).

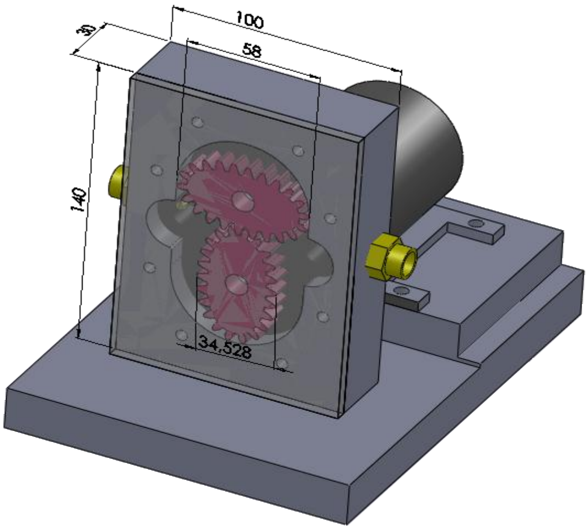

Figure 6. 3-D model the pump driven by elliptical spur gears

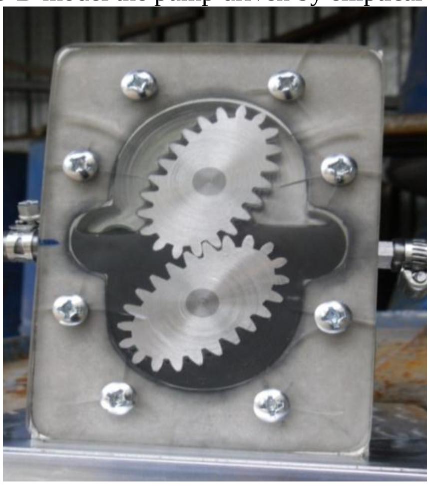

Figure 7. Elliptical spur gear pump produced

The flow rates of the pump produced were made at 7 different speeds by using Festo hydraulic circuit elements, DT-2236 Digital Photo/Contact tachometer and hydraulic motor oil.

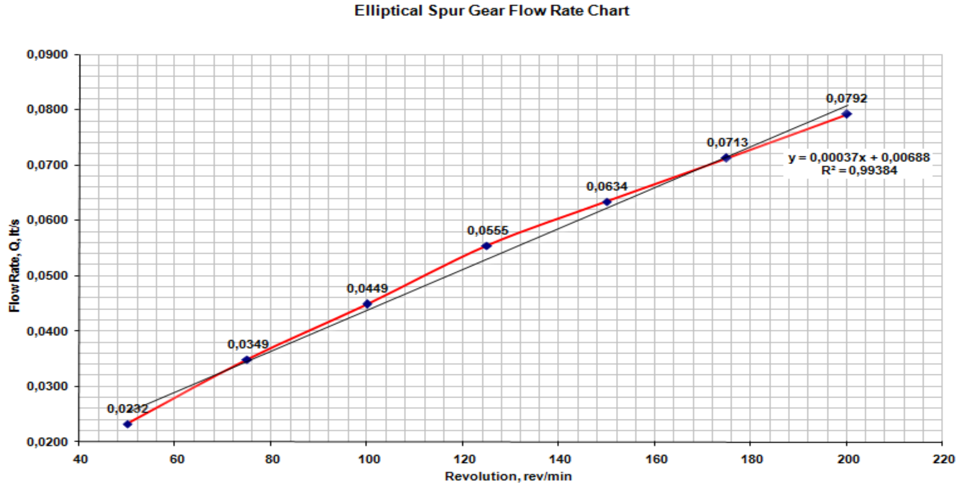

Figure 8. The flow rate graphics of elliptical gear pump

On the other hand, the pressure variations of the elliptical spur gear pump were made at 6 different speeds by using hydraulic circuit elements and pressure adjustment valve. The highest pressure was measured to be 10 bars at 175 rev/min, which is the highest speed the experimental mechanism provided. The graphics of the values obtained at pressure experiment and the equation of the linear line and R2 are given in Fig.9. and experimental study is shown in Fig.10.

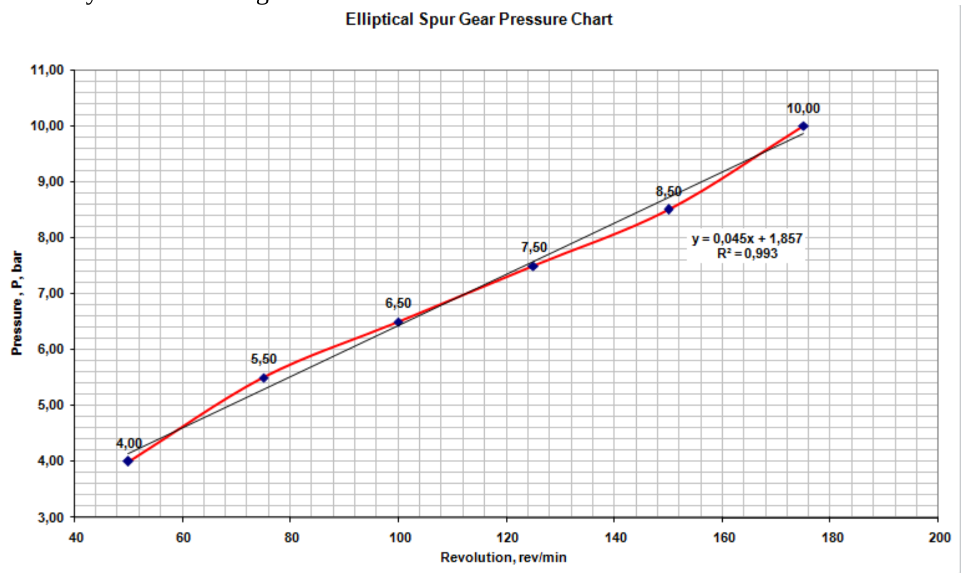

Figure 9. Pressure graphics of elliptical spur gear pump

Figure 10. The empirical pressure variations at elliptical spur gear pump

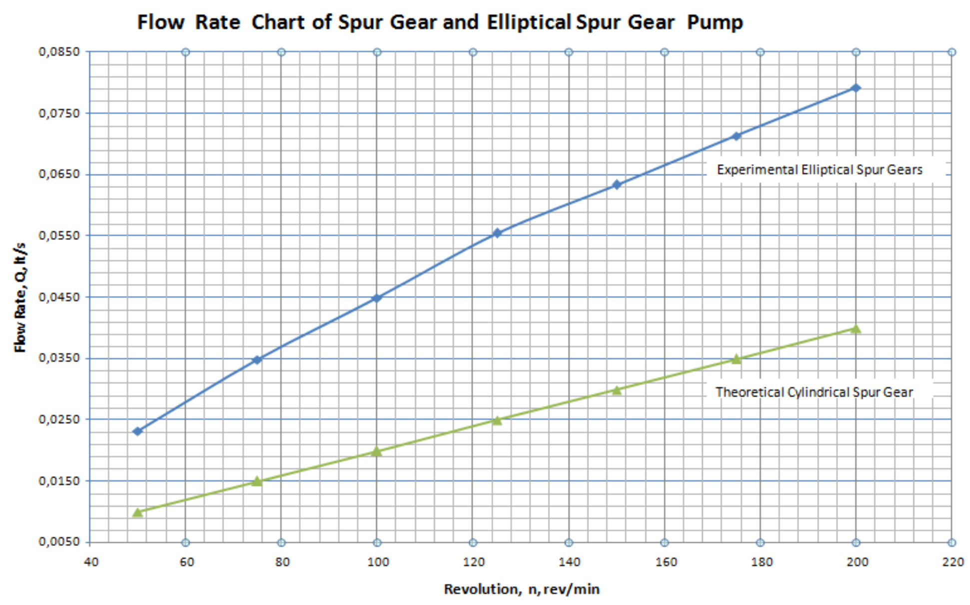

Figure 11. Flow rate changes at spur gear and elliptical gear pumps

Experiments on elliptical gear pumps produced were made at slow speeds between 50 rev/min and 200 rev/min. The reason for this is that the recent works on elliptical gear pumps have generally been on medicine, and especially on artificial hearts and kidney machines. On the other hand, elliptical gears have widely been used to measure high-speed flow rates in hydraulic systems. Additionally, flow rate calculation of the spur gear pump with 2-module and 22-teeth has been made within the context of this study and the results are given in Table 1. When the results of the elliptical gear pump are compared with those of the spur gear pump, it is observed that the elliptical gear hydraulic pump has more efficiency and the difference between increases as the cycle increases.

Table 1. Flow rate calculations of a pump cylindrical spur gears according to [29]

Spur gear elements		n (rev/min)	W(Rad/s)	Do	b	h	Qdd (lt/s)
b (cm)	1	50	5,235833	4,4	1	0,43	0,00998
M	2	75	7,85375	4,4	1	0,43	0,01497
h (cm)	0,4332	100	10,47167	4,4	1	0,43	0,01996
Z	22	125	13,08958	4,4	1	0,43	0,02495
Do (cm)	4,4	150	15,7075	4,4	1	0,43	0,02994
		175	18,32542	4,4	1	0,43	0,03493
		200	20,94333	4,4	1	0,43	0,03992

6. Conclusions

It has been observed that non-circular gears are not widely used at industry since their production and operation procedures are not well known. As a result few international firms produce this type of gears. However, non-circular gears have a number of advantages with respect to eccentricity and linkages. Non-circular gears can transfer bigger forces in comparison to eccentricities and their links can be balanced more easily and accurately. This is significant especially to the high-speed and highly-sensitive machines.

With this study a new approach has been presented to calculate the parameters of elliptical gear pumps. The calculations of curves elliptically rotating has been based on mathematical calculations of complex numbers.

The pair of elliptical gear with 2-module, 22-tooth and whose a/b = 1,778 has been placed into the hydraulic pump design. The main characteristic curves of the hydraulic pump have been obtained. With this experiment, it has been proved that elliptical gear pumps can work properly even at highly low speed rates and can be used at high flow rates in comparison with cylindrical spur gear pumps. This result is highly significant for artificial hearts and dialysis machines. It can be possible to produce more efficient elliptical spur gear pumps by using different materials, different gear dimensions.

The increase in the number of revolution will lead to the increase in the circular speed of elliptical spur gear, which will also results in an increase in the sound level and abrasion. Therefore, future research and experiments will be important on the surface quality, hardness, and abrasion of elliptical spur gears that work in high speeds. Also, it has been assessed that elliptical gears can measure extremely sensitive and high speed flow rates.

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